L-FUNCTIONS OF ELLIPTIC CURVES

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Let E be an elliptic curve over the field of rational numbers $\mathbb{Q}$ given by the minimal global Weierstrass equation:

$$E : y^2 + A_1xy + A_3y = x^3 + A_2x^2 + A_4x + A_6$$

and let Δ_E be its discriminant. For each prime p we put

$$a_p = p + 1 - \#E(F_p),$$

where $E(F_p)$ is the reduction of E modulo p. The L-function associated to E is given by

$$L(s, E) = \prod_{p|\Delta_E} \frac{1}{1 - a_p p^{-s}} \prod_{p\nmid\Delta_E} \frac{1}{1 - a_p p^{-s} + p^{1-2s}}.$$

The infinite product above is convergent for $Re(s) > 3/2$ and therefore we can expand it into a series $L(s, E) = \sum\limits_{n\geq 1} a_n n^{-s}$.

In this talk, we show that the set of positive integers n such that $|a_n|$ is a generalized Fibonacci number has asymptotic density 0.

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