Solving Diophantine Equations via Modular Curves

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- Diophantine equations define algebraic curves and algebraic surfaces.

Original Statement of FLT

1661-1665

'It is impossible to separate a cube into two cubes or a fourth power into fourth powers or, in general, any power greater than the second powers of like degree. I have discovered a truly marvelous demonstration, which this margin is too narrow to contain.'

Fermat's Last Theorem

Theorem (Wiles, Taylor-Wiles)

The equation

$$FLT_n : x^n + y^n = z^n$$

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First General Proof

By Mr. Le Blanc in 1823:

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Theorem

If p and $2p + 1$ are both prime, then $x^p + y^p = z^p$ has no solutions for which xyz is not divisible by p.

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1776-1831

"Monsieur Leblanc"

Germain & Gauss

Gauss:

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Gauss:

'...when a woman, because of her sex, our customs and prejudices, encounters infinitely more obstacles than men, yet overcomes these fetters and penetrates that which is most hidden, she doubtless has the most noble courage, extraordinary talent and superior genius. Nothing could prove to me in a more flattering and less equivocal way that the attractions of that science, which have added so much joy to my life, are not chimerical, than the favor with which you have honored it..'

...cinsiyetinden, geleneklerimiz ve önyargılarımızdan ötürü bir kadın, erkeklere oranla çok daha fazla engelle karşılaşıyor, yine de bu engelleri alt ediyor ve gizli saklı olana nüfuz edebiliyorsa, şüphesiz ki o çok asil bir cesarete, olağanüstü bir yeteneğe ve üstün bir dehaya sahiptir. Hiçbir şey bana, hayatıma büyük bir mutluluk katan o bilimin cazibesinin asılsız olmadığını, o cazibeyi şereflendirişinizden daha hoş ve kesin bir şekilde ispatlayamaz.

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Two dimensional surfaces in 3 dimensional space can be classified according to their genus.

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Granville and **Heath-Brown**: the number of solutions of FLT -if they exist- decreases as the exponent n increases, i.e. FLT is '*almost always*' true, if there are solutions they are few and very far between.

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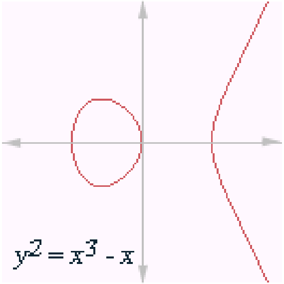
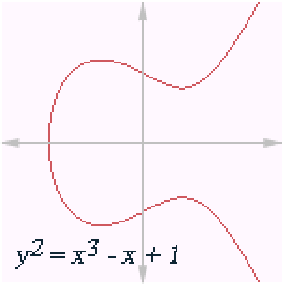
- Complex solutions of such a cubic form a torus.
- $E(\mathbb{Q}) = \{(x, y)|x, y \in \mathbb{Q}, y^2 = x^3 + ax + b\}$, $y^2 = x(x-1)(x+1)$ has only 4 points in $E(\mathbb{Q})$.

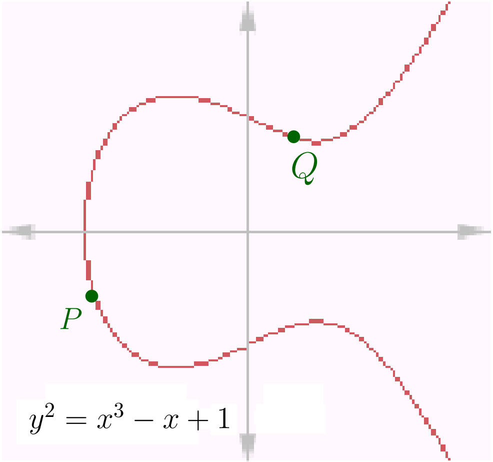




- These curves are not ‘closed’. We should ‘close them up’.
- This is done by adding another point to the curve, ‘*point at* ∞’. Denote this point as O.

Group Law on Elliptic Curves

We can ADD points on an elliptic curve, the point at ∞ is the *identity element*.



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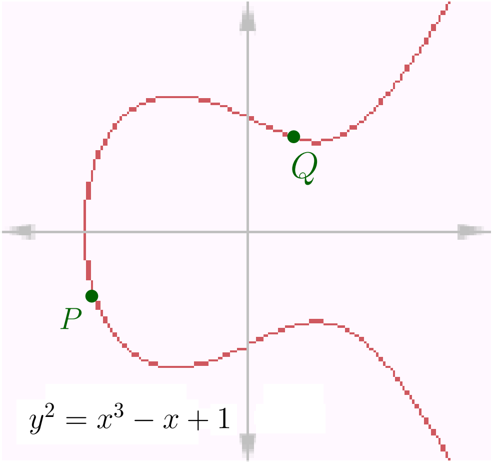
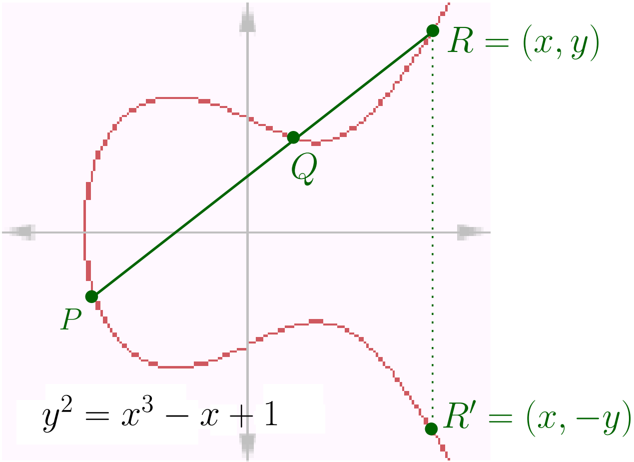
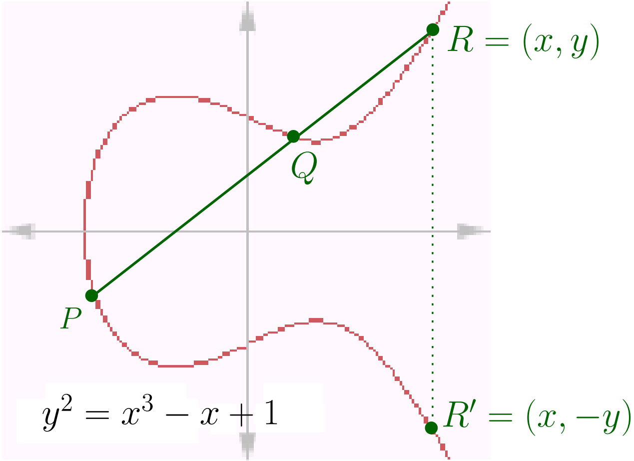



$R = (x, y)$
Q
P
$y^2 = x^3 - x + 1$
$R' = (x, -y)$



$$P \oplus Q = R'$$

Elliptic Curve mod p

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N_p	7	8	12	10	19	14	22	23	37	35	36	51

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- Hasse's Theorem: $|N_p - (p+1)| \leq 2\sqrt{p}$.

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- Can we predict a_p?
- For instance, is there a complex function $f(z)$ which has power series representation $\sum_{n=1}^{\infty} a'_n q^n$ where $q = \exp^{2\pi i}$, $a_p = a'_p$?

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+ 1$q^{23} - q^{25} - 9q^{27}$**- 7**$q^{29}$**- 3**$q^{31} - 6q^{33} + 8q^{35}$**+ 2**$q^{37} + 15q^{39}$**- 9**$q^{41} + \ldots$

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+ $\mathbf{1}q^{23} - q^{25} - 9q^{27}$**-** $\mathbf{7}q^{29}$**-** $\mathbf{3}q^{31} - 6q^{33} + 8q^{35}$**+** $\mathbf{2}q^{37} + 15q^{39}$**-** $\mathbf{9}q^{41} + \ldots$

Can we predict a_p? Yes, first observed by Eichler.

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- Modular forms on the complex plane have symmetries wrt the more complicated transformations $f(z) \mapsto f(\frac{az+b}{cz+d})$.
- Modular forms have power series representations i.e. they can be written as $\sum_{n=1}^{\infty} a'_n q^n$ where $q = \exp^{2\pi i}$.
- a MEC is an elliptic curve which can be 'parametrized' by a modular form. (The a_p's coming from the e.c. correspond to the coefficients of a modular form a'_p.)

Modularity Conjecture

Taniyama(55) and Shimura(57): Every elliptic curve over rational numbers is parametrized by a modular form.
OR Every elliptic curve over $\mathbb{Q}$ is modular.

Taniyama, 1927-1958

Shimura, 1930-

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by Wiles, Taylor-Wiles, Breuil, Conrad, Diamond, Taylor

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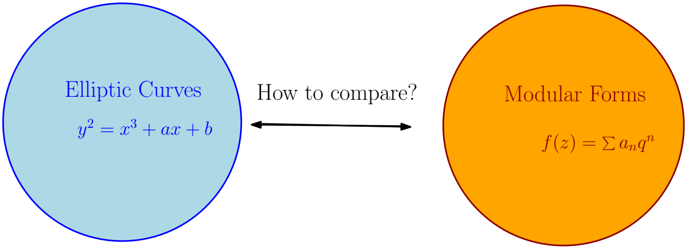
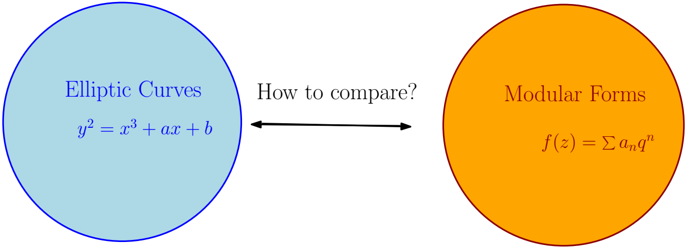


Comparing Different Worlds



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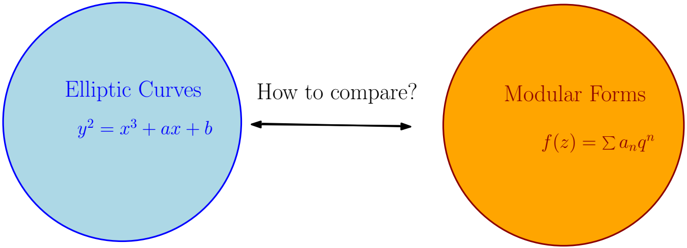

A third object is needed....

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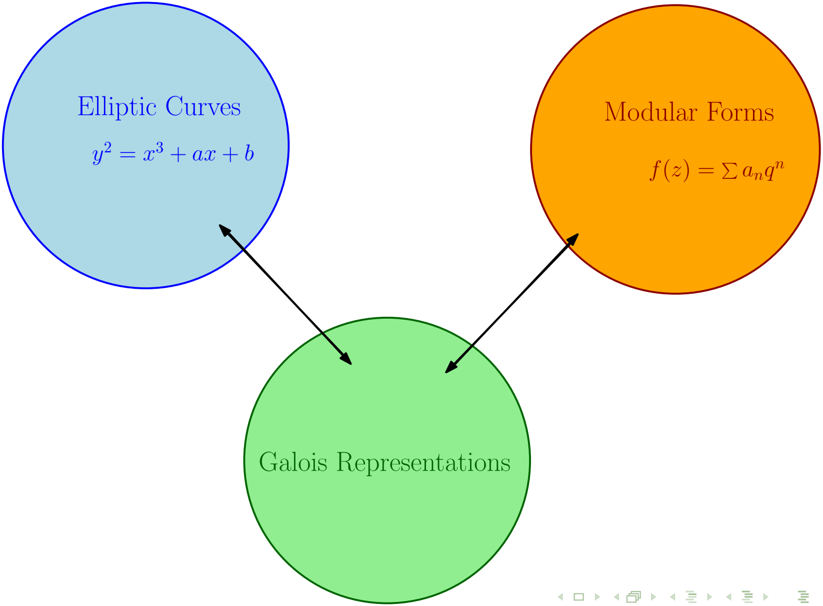

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- Elliptic Curves
- Modular Forms
- Galois Representations

How to use these ideas to solve Diophantine equations such as FLT?

Ribet's Level Lowering Theorem

Theorem

E elliptic curve over $\mathbb{Q}$. If E satisfies a technical condition then E corresponds to a special modular form (called newform) of level N_p.

Technical condition:= E `doesn't have an isogeny whose kernel has size` p `for a prime` $p \geq 5$

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- Given N_p there are only finitely many newforms of level N_p.

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Assume FLT has a solution and associate the solution to an elliptic curve E called the Frey curve if possible.

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Say a, b, c is a nontrivial solution to FLT_p, i.e. $a^p + b^p = c^p$ then

$$E_{a,b,c} : y^2 = x(x - a^p)(x + b^p)$$

is called Frey elliptic curve.

Frey elliptic curve has very strange properties.

- E satisfies the 'technical condition' i.e. E doesn't have any p-isogenies (by a theorem of Mazur)
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Therefore by Ribet's thm:

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$$E_{a,b,c} : y^2 = x(x - a^p)(x + b^p)$$

is called Frey elliptic curve.

Ribet's formula $\rightarrow N_p = 2$
Therefore by Ribet's thm:

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- $\Rightarrow$ But there is no such newform!
- $\Rightarrow$ Hence, contradiction, FLT doesn't have a solution (a, b, c).

Summary

Wiles was the person who did the important final work on the theorem by proving a form of the modularity conjecture needed to prove FLT, the entire enterprise was the work of many people. And it is all their contributions, taken together, which brought the final solution.

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$$\text{Kummer} \stackrel{\texttt{Ideals}}{\to} \text{Mazur} \xrightarrow{\texttt{Eisenstein Ideal}} \text{Frey} \xrightarrow{\texttt{Frey Curve}}$$

$$\text{Serre,Ribet,Taniyama,Shimura} \xrightarrow{\texttt{ST implies FLT}} \text{Wiles} \Rightarrow \text{FLT}$$

Barry Mazur:

‘Number theory produces, without effort, innumerable problems which have a sweet, innocent air about them, tempting flowers; and yet ... number theory swarms with bugs, waiting to bite the tempted flower-lovers who, one bitten, are inspired to excess of effort!. ’

(From number theory as gadfly)

Sayı teorisi, çok da çaba sarf etmeden, tatlı ve masum, sayısız problem üretir; gönülçelen çiçekler gibidir. Ama aynı sayı teorisi, baştan çıkmış çiçek severleri ısırmayı bekleyen böceklerle doludur. kişi bir kere ısırılmaya görsün, aşırı çaba sarf etmeye hevesli hale gelir.